AS
Mathematics
MPC1

Unit: Pure Core 1

Mark scheme

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–x EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown
Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, the principal examiner may suggest that we award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
	NO MISREADS ALLOWED IN THIS QUESTION			
(a)	$\frac{1+4\sqrt{7}}{5+2\sqrt{7}}\times\frac{5-2\sqrt{7}}{5-2\sqrt{7}}$	**M1**		
	(Numerator =) $5+20\sqrt{7}-2\sqrt{7}-56$	**A1**		at least this far
	(Denominator = $25+10\sqrt{7}-10\sqrt{7}-28$ **)** $=-3$	**B1**		must be seen as denominator
	Value $=\frac{-51+18\sqrt{7}}{-3}$ $=17-6\sqrt{7}$	**A1cso**	**4**	condone $-6\sqrt{7}+17$
(b)	$x\left(9\sqrt{5}-"their"6\sqrt{5}\right)="their"4\sqrt{5}$	**M1**		attempt to write each term as $k\sqrt{5}$ with **either** $2\sqrt{45}=6\sqrt{5}$ **or** $\sqrt{80}=4\sqrt{5}$
	$x\left(9\sqrt{5}-6\sqrt{5}\right)=4\sqrt{5}$	**A1**		**OE** must have equation
	$x=\frac{4}{3}$ or $x=1\frac{1}{3}$ or $x=1.\dot{3}$	**A1**	**3**	must be simplified to one of these
	Total		**7**	

(a) Condone multiplication by $5-2\sqrt{7}$ instead of $\times\frac{5-2\sqrt{7}}{5-2\sqrt{7}}$ for **M1 only if** subsequent working shows multiplication by both numerator and denominator – otherwise **M0**

For first **A1** each term must be evaluated correctly but may be seen in a grid
An error in the denominator such as $25+5\sqrt{7}-5\sqrt{7}-28=-3$ should be given **B0** and it would then automatically lose the final **A1cso**

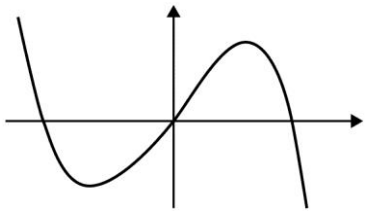
May use $\frac{1+4\sqrt{7}}{5+2\sqrt{7}}\times\frac{2\sqrt{7}-5}{2\sqrt{7}-5}$ **M1;** numerator = $-5-20\sqrt{7}+2\sqrt{7}+56$ **A1** etc

Alternative: $\frac{1+4\sqrt{7}}{5+2\sqrt{7}}=m+n\sqrt{7}$
leading to $5m+14n=1$; $2m+5n=4$ **M1** (one correct) **A1**(both correct)
either $m=17$ **or** $n=-6$ **A1;** **answer =** $17-6\sqrt{7}$ **A1 cso** (expression must be explicitly seen)

(b) **Alternative 1** Multiply or divide **each** term by $\sqrt{5}$ obtaining 3 integer terms (possibly with one error) **M1**
3 correct integer terms; eg $x(45-30)=20$ **A1** ; $x=\frac{4}{3}$ **A1**

Alternative 2 $x=\frac{\sqrt{80}}{9\sqrt{5}-2\sqrt{45}}\times\frac{9\sqrt{5}+2\sqrt{45}}{9\sqrt{5}+2\sqrt{45}}$ **M1** $=\frac{300}{225}$ **OE** **A1**(integer/integer) $=\frac{4}{3}$ **A1**

Squaring: Do not allow final **A1** unless justification for rejecting negative value
May earn **M1** for $x^2\left(k\sqrt{5}\right)^2=80$ and **A1** for $x^2=\frac{16}{9}$ **OE**

Q2	Solution	Mark	Total	Comment
(a)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\quad 20-2x-6x^2$	**M1** **A1**		two terms correct all correct
	$(10-6x)(2+x)\quad(=0)\qquad$ **OE**	**A1**		correct factors or correct use of formula for correct quadratic possibly multiplied by –1 or divided by ± 2
	(other stationary point when) $\quad x=\frac{5}{3}$	**A1cso**	**4**	**OE eg** $\frac{20}{12}$, $1\frac{2}{3}$, $1.\dot{6}$ but not $\frac{-20}{-12}$
(b)	$\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\right)\quad -2-12x$	**B1**		
	$\left(\text{when } x=-2,\ \frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)=(-2+24)=22$	**B1ft**		Sub $x=-2$ into their $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and evaluate correctly
	$22>0$ therefore **minimum** (point)	**E1ft**	**3**	**FT** their value of $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ but must have reason
(c)	Cubic graph **through origin** with one max & one min on either side of y-axis 	**M1** **A1**	**2**	may be reflection of given graph in x-axis for **M1** Graph roughly as shown in all 4 quadrants
	Total		**9**	

(a) If candidate multiplies by –1 before differentiating and writes $\frac{\mathrm{d}y}{\mathrm{d}x}=6x^2+2x-20$ this scores **M0**

For second **A1**, may have $(5-3x)(4+2x)$ or $2(3x-5)(x+2)$ or $(5-3x)(2+x)$ etc

Second **A1** is earned for formula as far as $\frac{2\pm 22}{-12}$ or $\frac{-1\pm 11}{6}$ etc

If both values given $x=-2,\ \frac{5}{3}$ then allow **A1cso**

Withhold **A1cso** if no **"=0"** seen or incorrect equating of expressions.

(b) Allow **E1ft** for "their –2+24 > 0 so min" etc or

$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}>0$ (provided they have a value earlier) $\Rightarrow$ minimum etc

(c) Allow **M1** if the curve does not actually cross the x-axis 3 times

For **A1** ignore any "numbers" on graph

Q3	Solution	Mark	Total	Comment
(a)	$(-2)^3 + b(-2)^2 + c(-2) + 24$	**M1**		clear attempt at p(–2)
	$-8 + 4b - 2c + 24 = 0$			
	$2b - c + 8 = 0$	**A1**	**2**	**AG** must see powers of –2 simplified correctly and = 0 appearing before last line
(b)	$3^3 + 3^2 b + 3c + 24 = -30$	**M1**		clear attempt at p(3) **and** = –30
	$27 + 9b + 3c + 24 = -30$ $3b + c + 27 = 0$	**A1**	**2**	**ACF** terms need not be collected but powers of 3 must be evaluated **No ISW -** mark their final equation
(c)	Correctly eliminating b or c from $2b - c + 8 = 0$ and an equation from **(b)**	**M1**		**PI** by one correct answer
	$b = -7$ or $c = -6$	**A1**		
	$b = -7$ **and** $c = -6$	**A1**	**3**	
	Total		**7**	

(a) Condone poor use of brackets if recovered on next line for both **M1** and **A1**
Note that "= 0" must appear before last line;
Example $p(-2) = -8 + 4b - 2c + 24 \Rightarrow 2b - c + 8 = 0$ scores **M1 A0**
M1 may also be earned for a full long division attempt by (x+2) as far as the remainder in terms of b and c.
M1 also for $p(x) = (x+2)(x^2 + dx + 12)$ $b = 2 + d$; $c = 2d + 12$ with **A1** for completion.
Terms must be exactly as printed answer but accept $0 = 2b - c + 8$ for **A1.**

(b) Do NOT treat use of 30 instead of –30 as a misread.
M1 may also be earned for a full long division attempt by (x–3) as far as the remainder **and** equating the remainder (in terms of b and c) to –30.

Ignore trailing equals sign for **A1** in both parts **(a)** and **(b).**

Q4	Solution	Mark	Total	Comment
(a)	$\text{grad } AB = \frac{-6-5}{8--2}$ or $\frac{5--6}{-2-8}$	**M1**		correct unsimplified; **PI** by $-\frac{11}{10}$
	$= -\frac{11}{10}$	**A1**		
	$y-5 = \text{"their"}-\frac{11}{10}(x--2)$ **or** $y--6 = \text{"their"}-\frac{11}{10}(x-8)$	**M1**		ft "their gradient" but must use *A* or *B* coordinates correctly
	$11x+10y=28$	**A1**	**4**	integer coefficients with *x* and *y* terms on one side and integer on other side
(b)	$(\text{grad } AC =)\frac{k-4}{k+2}$ **or** $(\text{grad } BC =)\frac{k+7}{k-8}$	**B1**		correct simplified expression eg $\frac{4-k}{-2-k}$
	$\frac{k-4}{k+2}\times\frac{k+7}{k-8}=-1$ **OE**	**M1**		condone one error in one term
	$k^2+3k-28=-k^2+6k+16$ $\Rightarrow 2k^2-3k-44\ (=0)$	**A1**		
	$(k+4)(2k-11)\quad (=0)$	**dM1**		***attempt*** at factorisation or correct use of formula for "their" quadratic
	$k=-4,\quad k=5\frac{1}{2}$ **OE**	**A1**	**5**	
	Total		**9**	

(a) May earn second **M1** for $y = \text{"their"}-\frac{11}{10}x+c$ and attempt to find *c* using $x=-2, y=5$ or $x=8, y=-6$

For **A1 ,** condone any equivalent equation of the correct form such as $28=10y+11x$ **or** $-22x-20y=-56$ but **not** $11x+10y-28=0$ or any equation with non-integer coefficients

(b) **Alternative** $\lfloor AC^2 = \rfloor (k-4)^2+(k+2)^2$ **or** $\lfloor BC^2 = \rfloor (k-8)^2+(k+7)^2$ **B1** (may be under square root)

Pythagoras $(k-4)^2+(k+2)^2+(k-8)^2+(k+7)^2=10^2+11^2$ **M1** (condone one error in one term)

$k^2-8k+16+k^2+4k+4+k^2-16k+64+k^2+14k+49=221 \Rightarrow 4k^2-6k-88=0$ **A1 etc**

Allow **dM1** for factors that would multiply out to give their k^2 and constant terms.

To earn **dM1** using formula then it must be correct for "their" quadratic and discriminant evaluated "correctly" **;** if correct quadratic used, you must see $\frac{3\pm\sqrt{361}}{4}$ **OE** to award **dM1**

Q5	Solution	Mark	Total	Comment
(a)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\ 8x^3-9x^2$	**M1** **A1**		one term correct $\frac{\mathrm{d}y}{\mathrm{d}x}$ correct
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=8(-1)^3-9(-1)^2\right)\ =-17$	**dM1**		correct substitution of $x=-1$ and correct evaluation for "their" $\frac{\mathrm{d}y}{\mathrm{d}x}$
	(Grad of normal =) $\frac{1}{17}$	**A1ft**		**FT** their negative reciprocal provided **M1** is earned
	$y=\frac{1}{17}x+9\frac{1}{17}$	**A1**	**5**	**or** $y=\frac{1}{17}x+\frac{154}{17}$ **equation must be in this form**
(b)(i)	$\frac{2}{5}x^5-\frac{3}{4}x^4+4x$	**M1** **A1**		two terms correct all correct (may have +c)
	$\left[\frac{2}{5}\times2^5-\frac{3}{4}\times2^4+4\times2\right]-$ $\left[\frac{2}{5}\times(-1)^5-\frac{3}{4}\times(-1)^4+4\times(-1)\right]$	**dM1**		correct substitution of 2 and –1 to find "their" F(2) – F(–1)
	$\left[\frac{2\times32}{5}-\frac{3\times16}{4}+8\right]-\left[-\frac{2}{5}-\frac{3}{4}-4\right]$	**A1**		**correct** with powers of 2 and (–1) and minus signs handled correctly
	= $13\frac{19}{20}$ or 13.95 or $\frac{279}{20}$	**A1**	**5**	correct single equivalent fraction or decimal
(ii)	Area of trapezium = $\frac{1}{2}(9+12)\times3$ = 31.5 or $\frac{63}{2}$ **OE**	**B1**		or $\int_{-1}^{2}(x+10)\,\mathrm{d}x=31.5$
	"their" trapezium – answer from **(b)(i)**	**M1**		must be trapezium
	(Area of shaded region =**)** 31.5 – 13.95 = 17.55	**A1**	**3**	**OE** $17\frac{11}{20}$, $\frac{351}{20}$
	Total		**13**	

(a) Do **not** accept $y=\frac{1}{17}x+c,\ldots\quad c=\frac{154}{17}$ for final **A1;** equation must be written in full on one line.

Allow $y=\frac{x}{17}+\frac{154}{17}$ for final **A1 but not** $y=\frac{x+154}{17}$ **.**

(b)(i) Use of F(–1) – F(2) scores **dM0**

(ii) For **M1** condone use of their"**(b)(i)** – "their" trapezium"**.**
Be generous in awarding this **M1** provided you are convinced they are considering the area of a trapezium.

Q6	Solution	Mark	Total	Comment
(a)	$(x+10)^2+(y-7)^2$ **=…**	**M1**		one of these terms correct
		A1		LHS correct ignoring any extra constants **or** $(x--10)^2+(y-7)^2$ **=…**
	$(x+10)^2+(y-7)^2=10^2$ **(**or …=100)	**A1**	**3**	**or** $(x--10)^2+(y-7)^2=10^2$ **(**or …=100)
(b)	$10^2+(y-7)^2=10^2$ $\Rightarrow (y-7)^2=0 \quad \Rightarrow y=7$	**M1**		putting x=0 in "their" equation and attempt to solve for y
	Repeated root means circle touches y-axis	**E1**		completely correct working and **both parts** of the conclusion
	$(x+10)^2+7^2=100$ $(x+10)^2=51 \quad \Rightarrow x=-10\pm\sqrt{51}$	**M1**		putting y = 0 in "their" equation and attempt to solve for x
	Two roots so circle crosses x–axis twice	**E1**	**4**	completely correct working and **both parts** of the conclusion
(c)(i)	$(x+10)^2+(kx-5)^2=100$ $x^2+20x+100+k^2x^2-10kx+25=100$	**M1**		sub $y=kx+2$ into "their" circle equation and attempt to multiply out brackets
	$(1+k^2)x^2+10(2-k)x+25=0$ must have terms exactly as printed answer	**A1cso**	**2**	**AG** be convinced - condone $0=(1+k^2)x^2+10(2-k)x+25$
(ii)	$10^2(2-k)^2-4\times25(1+k^2)$	**M1**		correct discriminant unsimplified
	$400-400k+100k^2-100-100k^2\ (=0)$	**A1**		multiplying out correctly
	$k=\frac{3}{4}$	**A1cso**	**3**	must see "=0" before final answer
	Total		**12**	

(b) May have "their" $y^2-14y+49=0$ and ***attempt at*** formula or discriminant for first **M1**
with **E1** awarded for being *completely correct* with ***both parts of conclusion*** such as " only one value of y / root/ solution so touches y–axis" or "discriminant = 0 therefore repeated root so touches y–axis"
Second **M1** can be earned for "their" $x^2+20x+49=0$ and ***attempt at*** formula or discriminant with **E1** awarded for being *completely correct* with ***both parts of conclusion*** such as " two (different) values of x / roots/ solutions so crosses x-axis in two places"

May use geometry : award first **M1** for stating x-coord of centre = "their" –10 and radius ="their" 10; then **E1** for **both** stating that ***lengths are equal*** hence circle touches y-axis (or y-axis is a tangent to the circle)
Second **M1** for stating y-coord of centre = "their" 7 and radius ="their" 10; **E1** for **both** explaining that $y_C < r$ and concluding statement that the circle crosses x-axis at two distinct points.
To award each of these **E1** marks all working must be correct.

(c)(i) Penalise trailing equals signs and any incorrect algebra even if recovered awarding **A0cso**
All terms must be exactly as in the printed answer but "0=" may be on the LHS for **A1cso**.
May use $x^2+(kx+2)^2+20x-14(kx+2)+49=0$ and attempt to multiply out brackets for **M1** giving $x^2+k^2x^2+4kx+4+20x-14kx-28+49=0$ leading to $(1+k^2)x^2+10(2-k)x+25=0$ for **A1cso**

(ii) May write $10^2(2-k)^2=4\times25(1+k^2)$ for **M1** then $4-4k+k^2=1+k^2$ for **A1**
Correct discriminant appearing within the quadratic equation formula can earn **M1**.

Q7	Solution	Mark	Total	Comment
(a) (i)	$x+x+(x+y)+(x+y)=15$ **OE**	**M1**		**ACF** eg $2x+2(x+y)=15$
	$y=\frac{1}{2}(15-4x)$ **OE**	**A1**	**2**	
(ii)	$[S=]\ (x+y)^2-y^2$ **OE**	**M1**		$[S=]x^2+2xy$ **or** $x(x+y)+xy$ etc
	$=x^2+x(15-4x)$	**dM1**		Sub their y into correct S expression
	$=15x-3x^2$			
	$S=3(5x-x^2)$	**A1**	**3**	**AG** be convinced
(b)(i)	$A-(x-2.5)^2$	**M1**		must have both minus signs but A may be negative but not zero
	$6.25-(x-2.5)^2$	**A1**	**2**	$p-(x-q)^2$ **;** $p=6.25,\quad q=2.5$ **OE** $\frac{25}{4}-\left(x-\frac{5}{2}\right)^2$
(ii)	**(**Max $S=$**)** $3\times$"*their*" p (must be > 0)	**M1**		**or** $3(5\times$"*their*" $q-$"*their*" $q^2)$
	$=$ 18.75	**A1**	**2**	**OE** eg $\frac{75}{4}$ withhold **A1** if 2 marks not scored in **(b)(i)**
	Total		**9**	

(a)(i) Penalise candidates who are clearly working back from printed answer in part **(a)(ii)**

(ii) Only allow **dM1** if "their y" is of the form $a+bx$;
expression need not be simplified so $S=(x+7.5-2x)^2-(7.5-2x)^2$, for example, earns **dM1**.
Must have "$S=$…" for final **A1** but allow $3(5x-x^2)=S$ **;** condone "S m$^2=$…" but not "$S^2=$…"
Allow omission of "$S=$…" on final line provided it appears on an earlier line with "=" on all subsequent lines.

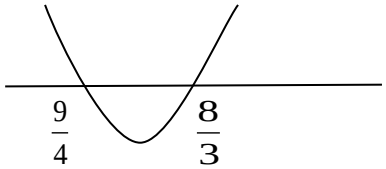
(b)(i) **Example 1** $-(x-2.5)^2+\frac{25}{4}$ scores **M1 A1 (**and **ISW** any incorrect rearrangement)

Example 2 $-(x-2.5)^2-\frac{25}{16}$ scores **M1 A0**

Alternative for **M1**: $x^2-5x=(x-2.5)^2-2.5^2$ **but** must have both sides of this identity

If **M0** is scored then award **SC B1** for $6.25-(x-2.5)$ **or** $6.25-(2.5-x)^2$

(ii) Allow **SC B1** if 18.75 is clearly obtained via differentiation or **NMS**

Q8	Solution	Mark	Total	Comment
(a)	$\left(\frac{\mathrm{d}h}{\mathrm{d}t}=\right)\ 12t^2-59t+72$ **OE**	**M1** **A1**		two terms correct (may have x for t) all correct – must have t
	$t=3\Rightarrow\left(\frac{\mathrm{d}h}{\mathrm{d}t}=\right)12\times3^2-59\times3+72$	**dM1**		substituting t = 3 into their $\frac{\mathrm{d}h}{\mathrm{d}t}$
	$(108-177+72=)\ 3$	**A1**	**4**	
(b)	(Decreasing) $\Rightarrow 12t^2-59t+72<0$	**B1ft**		**FT** their $\frac{\mathrm{d}h}{\mathrm{d}t}$ but must have " < 0 "
	$(4t-9)(3t-8)$	**M1**		attempt at factors or correct use of formula
	CVs are $(t=)\frac{9}{4},\ (t=)\frac{8}{3}$	**A1**		
	+ – + $\frac{9}{4}$ $\frac{8}{3}$	**M1**		use of sign diagram or sketch 
	$\frac{9}{4}<t<\frac{8}{3}$	**A1**	**5**	fractions must be simplified & **B1** earned for final **A1**
	may have $t<\frac{8}{3}$ **AND** $t>\frac{9}{4}$			no **ISW** here
	Total		**9**	

(b)

For first **M1**, if using formula need to go as far as $\frac{59\pm\sqrt{25}}{24}$ if inequality is correct and to a similar form if following through from their quadratic; if factorising quadratic (correct or incorrect) then factors should multiply out to give t^2 and constant terms to earn first **M1**.

For second **M1**, if critical values are correct then sign diagram or sketch must be correct ***with correct CVs marked in correct order***.
However, if CVs are not correct then second **M1** can be earned for attempt at sketch or sign diagram but ***their CVs*** MUST be marked correctly on the diagram or sketch.
For final **A1,** inequality must have t and no other letter.

If **B1** is earned and correct quadratic inequality is seen,

final answer of $\frac{9}{4}<t<\frac{8}{3}$ (with or without working) scores final **4 marks**.

(A) $\frac{9}{4}<x<\frac{8}{3}$ (B) $t<\frac{8}{3}$ **OR** $t>\frac{9}{4}$ **(**C) $t<\frac{8}{3}$, $t>\frac{9}{4}$ (D) $\frac{9}{4}$,, t,, $\frac{8}{3}$ (E) $\frac{54}{24}<t<\frac{64}{24}$ **OE**
with or without working, each score **3 marks** (**SC3**)
If **B1** is **NOT** earned, then only the next 3 marks are available, namely **M1, A1, M1** and **A0** even if final inequality is correct.

Example NMS $t<\frac{8}{3}$, $t<\frac{9}{4}$ scores **M1 A1 M0 (**since both CVs are correct)